Physics Qualifying Examination UCI May 17, 1976

GENERAL PHYSICS

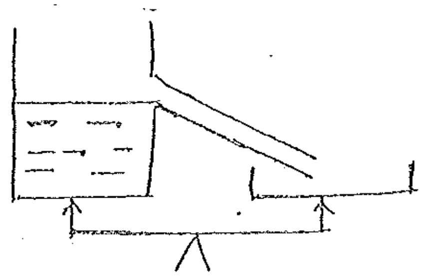
Do 8 out of 10.

1. Give numerical values for
   a) mass of: electron, proton, earth, sun, our galaxy.
   b) size of: classical electron radius, nuclear diameter, first Bohr orbit, distance to sun, distance to nearest star.
   c) time scale for: decay of an excited nucleus, decay of excited atom, half-life of carbon-14, half-life of uranium, size of earth, age of universe.
   d) energy required to: dissociate a molecule, ionize an atom, disrupt a nucleus, break parton into its component quarks?

2. Complete the listed nuclear and particle reactions:
   a) $p + e^- \rightarrow$ ____ $+ \nu_e$
   b) $\pi^+ \rightarrow \mu^+ +$ ____
   c) $\mu^+ \rightarrow$ ____ $+ \nu_e + \bar{\nu}_\mu$
   d) $C^{12} + He^4 \rightarrow$ ____
   e) $Li^7 + p \rightarrow He^4 +$ ____
   f) $C^{13} + d \rightarrow C^{14} +$ ____

3. An electron with velocity $\vec{v}$ close to speed of light is injected into a magnetic field of strength $\vec{B}$.
   a) What path will it follow?
   b) What is the emitted radiation called?
   c) In which direction is the radiation emitted?
   d) A fixed observer will see the radiation as a series of pulses. Why?
   e) What is the range of frequencies comprising the radiation?

4. Consider the device shown in the sketch. The tank on the left has a side arm. The water level is at the bottom edge of this arm. Add a bit of ice to the left tank. Is the balance preserved? Explain your answer.



5. The "arrow of time" is said to be related to the existence of irreversible phenomena. Elucidate.

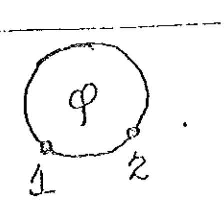
6. a) A stone is thrown from a boat into the swimming pool in which the boat is floating. Does the water level rise, fall, or remain the same? Explain briefly.
b) What happens to the level if a hole is made in the boat and it sinks?

7. A coin with an off center hole is spun. Does the hole migrate to the top, side, or bottom? Explain.

8. What is a boson? A fermion? Give an example of each. Describe the connection with statistics.

9. What is: a) Olber's paradox? How can it be resolved?
b) The density of a neutron star?
c) Mach's principle?

10. Given a conducting ring threaded by a changing magnetic flux, what is the potential difference between points 1 and 2?



Physics Qualifying Examination UCI May 19, 1976

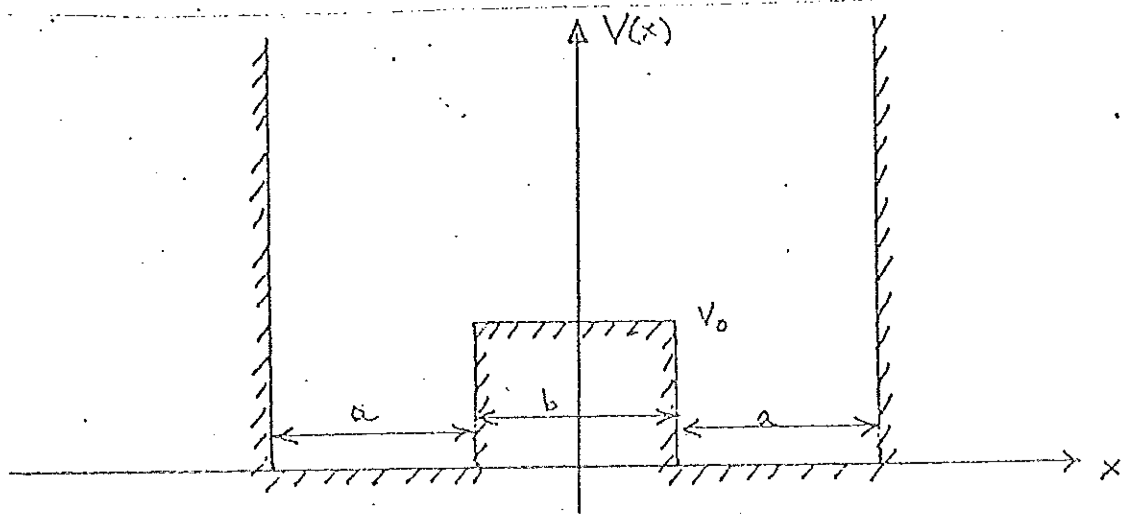
QUANTUM MECHANICS II

Do all the problems.

1. An electron is in the ground state of tritium, ^3H. A beta decay reaction, $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$ occurs, changing the ^3H nucleus to ^3He. Calculate the probability that the atomic electron will be in the ground state of $^3He^+$. (Hint: as an approximation, ignore the interaction between the beta decay electron and the atomic electron.)

2. Calculate the Zeeman structure and sketch the energy levels of an atom in the $^4D_{5/2}$ state which is immersed in a weak magnetic field.

3. Given the potential energy shown, sketch the wave functions and energy levels for $E < V_o$ and $E > V_o$.

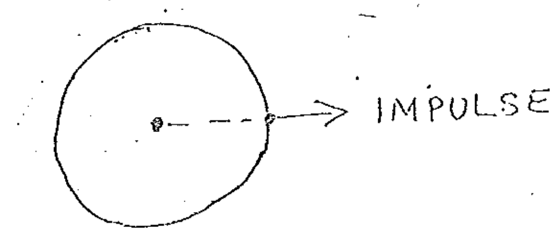


Discuss the effect on adjacent energy levels as either V_o or b becomes very large.

4. The eigenfunctions of spin $\frac{1}{2}$ are α and β which are the usual up and down eigenfunctions of the z component of the spin operator, i.e., $\hat{S}_z$. Given a system of 3 spin $\frac{1}{2}$ particles, what are the eigenfunctions of spin 3/2? Give your answer in terms of the α's and β's mentioned above.

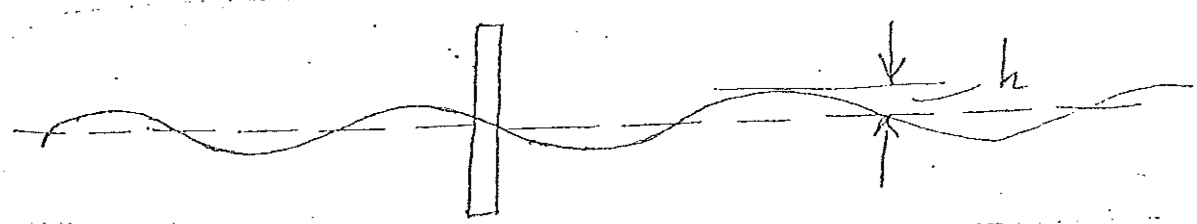
CLASSICAL MECHANICS: Do all four problems.

1. A spaceship is orbiting a planet in a circular orbit. It is given a weak, impulsive radial blow at sometime, as shown in the sketch below.



Find the form of the new orbit, and sketch its shape.

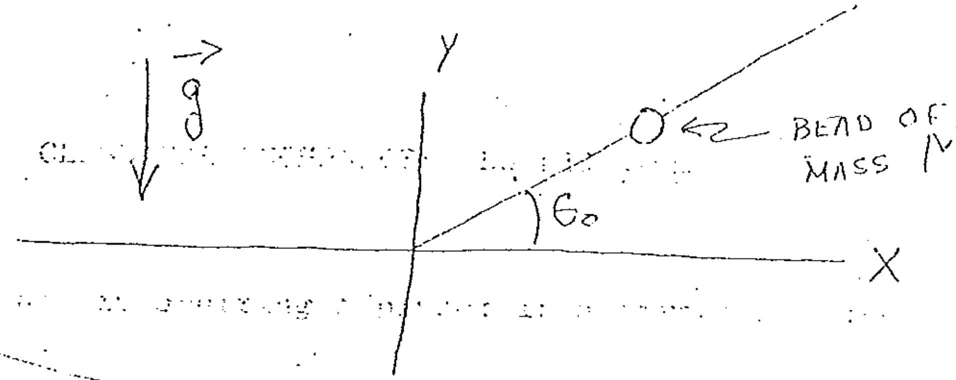
2. A stick of length ℓ floats in water, and gravity acts downward. The density of the water is ρ, and the density of the material of which the stick is made is $\frac{1}{2}\rho$. See sketch.



(a) Calculate the frequency of small oscillations of the stick, if the stick bobs up and down in calm water.

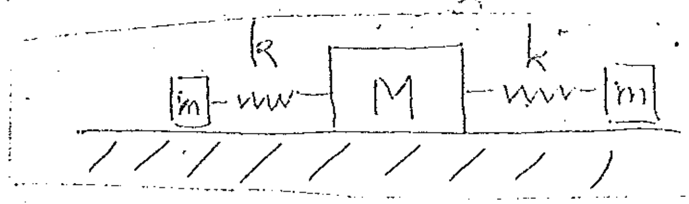
(b) A sinusoidal wave with frequency ω and height h passes by the stick. Calculate the amplitude of the stick as a function of ω, and sketch the result.

3. A bead of mass M slides without friction on a long straight wire inclined at angle θ_o with respect to the x axis. A gravitational field of strength g acts downward, and the wire is rotated about the y direction with angular velocity Ω_o (see sketch on next page).



Solve for the motion of the bead as a function of time, if it is presumed to be located at $x = x_o$ at time zero, with zero velocity along the wire. Describe the subsequent motion of the bead, for various possible values of x_o.

4. Two masses, each of mass m, and a third with mass M are hooked together by identical springs, as shown below. Each spring has force constant k, and the masses sit on a frictionless floor.



(a) Find the normal mode frequencies of the system, and sketch the motion associated with each frequency.

(b) The right hand spring is replaced by one with spring constant $k' > k$. Without performing a calculation, indicate which normal mode frequencies change, and sketch the motion of the masses for each normal mode of the new system. The sketch should emphasize any differences with the sketches in (a), and motivate the sketch with brief comments on the physical reasons why the change occurs as you draw it.

MATHEMATICAL PHYSICS

1. a) Evaluate the integral $\int_0^\infty \frac{x^2 dx}{x^4+1}$.

b) Evaluate the integral $\int_0^\infty \frac{\ell n x}{x^2+b^2}\, dx$.

(Hint: Make use of the function $\frac{(\ell n z)^2}{z^2+b^2}$.)

2. An anharmonic, one-dimensional oscillator is driven by an external cosinusoidal force, so that its equation of motion is

$$m\ddot{x} = -kx - \lambda x^2 + F_o \cos(\omega t)$$

Among the consequences of the non-linear term in this equation is that the amplitude x(t) acquires a D.C. component, as well as a component that oscillates at frequency 2ω. Find the expressions for each of these two components, to lowest order in the small quantity λ.

3. A square membrane of side L is confined to a rigid frame, and it vibrates with amplitude $u(\vec{x},t)$ that obeys the equation

$$T\nabla^2 u - \rho \frac{\partial^2 u}{\partial t^2} - k \frac{\partial u}{\partial t} = f(t)\,\delta(x - x_o)\,\delta(y - y_o) \ .$$

Find a formal expression for the amplitude $u(\vec{x},t)$.

4. A function f(x) is represented in the closed interval [0,1] by the Fourier series

$$f(x) = \sum_{n=-\infty}^{+\infty} f_n \exp[2\pi i n x] \ .$$

Under what conditions are the Fourier components of df/dx given by $2\pi i n\, f_n$?

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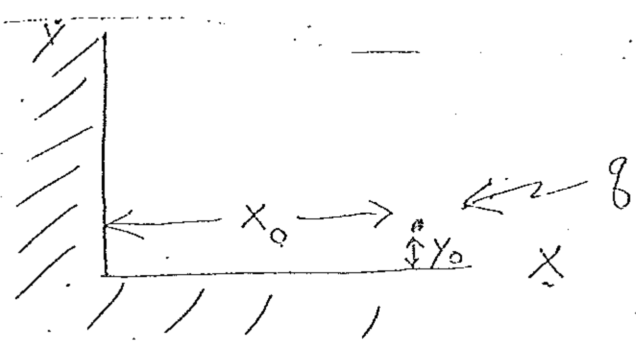
ELECTRICITY AND MAGNETISM

There are three problems. Complete all three.

1. Derive the equation for the resistance/unit length between two cylindrical electrodes of diameter a, separated by the distance ℓ. The electrodes are immersed in a fluid of conductivity σ, and assume $a \ll \ell$.

2. A spherical shell (very thin) has a uniform surface charge density σ spread over its surface. It spins about the z axis with angular velocity Ω_o.
   - (a) Find expressions for the electric and magnetic fields everywhere.
   - (b) Calculate the Poynting vector, and draw a sketch of of the lines of $\vec{S}$.
   - (c) Calculate the total angular momentum density stored in the fields, and the total angular momentum.

3. A charge + q is placed near the corner of two intersecting, grounded and perfectly conducting planes, as shown in the sketch:



Assume that $y_o \ll x_o$. The charge oscillates in the y direction, with small amplitude $y(t) = \Delta \cos(\omega t)$, where the frequency ω is small, in the sense that $\omega \ll c/x_o$. Find a formula for the Poynting vector far from the corner.

QUANTUM MECHANICS I

Do all the problems.

1. Suppose a parameter α occurs in the Hamiltonian H. It might be e.g. the range of the potential. Show that for a stationary, normalized state ψ that

$$\frac{\partial E}{\partial \alpha} = \frac{\partial}{\partial \alpha} \langle \psi H \psi \rangle = \langle \psi \frac{\partial H}{\partial \alpha} \psi \rangle$$

2. Given the harmonic oscillator Hamiltonian

$$H_o = \frac{p^2}{2m} + \tfrac{1}{2} m\omega^2 x^2$$

The matrix element $x_{n,n+1} = \langle n|x|n+1\rangle = \beta\left(\frac{n+1}{2}\right)^{\frac{1}{2}}$

where $\beta = \left(\frac{\hbar}{m\omega}\right)^{\frac{1}{2}}$.

a) Calculate $(x^3)_{n,n+1}$ and $(x^3)_{n,n+3}$

b) Given the perturbation $V = \alpha x^3$, calculate the perturbed energy of the oscillator ground state to second order.

3. Calculate a matrix representation of the spin operators $\hat{S}_z$, $\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y$ for the case $S = 2$.

4. A positron with velocity v, such that $c \gg v \gg e^2/\hbar$ is incident on a hydrogen atom in the ground state. Calculate the angular scattering cross section for the situation where where the hydrogen atom remains in the ground state. Justify all your approximations.

5. Given that the following squared matrix element

$$\left|\langle \alpha, J = \tfrac{3}{2}, \ J_z = \tfrac{1}{2} \left| Y_{\ell=2}^{m_\ell=0} \right| \beta, J = \tfrac{1}{2}, \ J_z = \tfrac{1}{2}\rangle\right|^2 = 17$$

Calculate

$$\left|\langle \alpha, J = \tfrac{3}{2}, J_z = -\tfrac{1}{2} \left| Y_{\ell=2}^{m_\ell=-1} \right| \beta, J = \tfrac{1}{2}, J_z = \tfrac{1}{2}\rangle\right|^2$$

and

$$\left|\langle \alpha, J = \tfrac{3}{2}, J_z = \tfrac{1}{2} \left| Y_{\ell=2}^{m_\ell=-1} \right| \beta, J = \tfrac{1}{2}, J_z = \tfrac{1}{2}\rangle\right|^2$$

6. Prove that trace $\gamma_\mu A^\mu \gamma_\sigma B^\sigma = 4A \cdot B$ where the γ's are the Dirac matrices.

CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A $\sqrt{\ }$ is to be understood over every coefficient: e.g., for -8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2			
m_1	m_2	Coefficients		
.	.			
.	.			

1/2 × 1/2

m_1 m_2	J=1	J=0
M	+1	
+1/2 +1/2	1	
M	0	0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2
M	−1	
−1/2 −1/2	1	

1 × 1/2

m_1 m_2	J=3/2	J=1/2
M	+3/2	
+1 +1/2	1	
M	+1/2	+1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3
M	−1/2	−1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3
M	−3/2	
−1 −1/2	1	

2 × 1/2

m_1 m_2	J=5/2	J=3/2
M	+5/2	
+2 +1/2	1	
M	+3/2	+3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5
M	+1/2	+1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5
M	−1/2	−1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5
M	−3/2	−3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5
M	−5/2	
−2 −1/2	1	

3/2 × 1/2

m_1 m_2	J=2	J=1
M	+2	
+3/2 +1/2	1	
M	+1	+1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4
M	0	0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2
M	−1	−1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4
M	−2	
−3/2 −1/2	1	

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2 × 1

m_1 m_2	J=3	J=2	J=1
M	+3		
+2 +1	1		
M	+2	+2	
+2 0	1/3	2/3	
+1 +1	2/3	−1/3	
M	+1	+1	+1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	6/15	−1/2	1/10
M	0	0	0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10
M	−1	−1	−1
0 −1	6/15	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5
M	−2	−2	
−1 −1	2/3	1/3	
−2 0	1/3	−2/3	
M	−3		
−2 −1	1		

3/2 × 1

m_1 m_2	J=5/2	J=3/2	J=1/2
M	+5/2		
+3/2 +1	1		
M	+3/2	+3/2	
+3/2 0	2/5	3/5	
+1/2 +1	3/5	−2/5	
M	+1/2	+1/2	+1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6
M	−1/2	−1/2	−1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2
M	−3/2	−3/2	
−1/2 −1	3/5	2/5	
−3/2 0	2/5	−3/5	
M	−5/2		
−3/2 −1	1		

1 × 1

m_1 m_2	J=2	J=1	J=0
M	+2		
+1 +1	1		
M	+1	+1	
+1 0	1/2	1/2	
0 +1	1/2	−1/2	
M	0	0	0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3
M	−1	−1	
0 −1	1/2	1/2	
−1 0	1/2	−1/2	
M	−2		
−1 −1	1		

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

3/2 × 3/2

m_1 m_2	J=3	J=2	J=1	J=0
M	+3			
+3/2 +3/2	1			
M	+2	+2		
+3/2 +1/2	1/2	1/2		
+1/2 +3/2	1/2	−1/2		
M	+1	+1	+1	
+3/2 −1/2	1/5	1/2	3/10	
+1/2 +1/2	3/5	0	−2/5	
−1/2 +3/2	1/5	−1/2	3/10	
M	0	0	0	0
+3/2 −3/2	1/20	1/4	9/20	1/4
+1/2 −1/2	9/20	1/4	−1/20	−1/4
−1/2 +1/2	9/20	−1/4	−1/20	1/4
−3/2 +3/2	1/20	−1/4	9/20	−1/4
M	−1	−1	−1	
+1/2 −3/2	1/5	1/2	3/10	
−1/2 −1/2	3/5	0	−2/5	
−3/2 +1/2	1/5	−1/2	3/10	
M	−2	−2		
−1/2 −3/2	1/2	1/2		
−3/2 −1/2	1/2	−1/2		
M	−3			
−3/2 −3/2	1			

$$d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2} \qquad d^1_{0,0} = \cos\theta$$

2 × 3/2

m_1 m_2	J=7/2	J=5/2	J=3/2	J=1/2
M	+7/2			
+2 +3/2	1			
M	+5/2	+5/2		
+2 +1/2	3/7	4/7		
+1 +3/2	4/7	−3/7		
M	+3/2	+3/2	+3/2	
+2 −1/2	1/7	16/35	2/5	
+1 +1/2	4/7	1/35	−2/5	
0 +3/2	2/7	−18/35	1/5	
M	+1/2	+1/2	+1/2	+1/2
+2 −3/2	1/35	6/35	2/5	2/5
+1 −1/2	12/35	5/14	0	−3/10
0 +1/2	18/35	−3/35	−1/5	1/5
−1 +3/2	4/35	−27/70	2/5	−1/10
M	−1/2	−1/2	−1/2	−1/2
+1 −3/2	4/35	27/70	2/5	1/10
0 −1/2	18/35	3/35	−1/5	−1/5
−1 +1/2	12/35	−5/14	0	3/10
−2 +3/2	1/35	−6/35	2/5	−2/5
M	−3/2	−3/2	−3/2	
0 −3/2	2/7	18/35	1/5	
−1 −1/2	4/7	−1/35	−2/5	
−2 +1/2	1/7	−16/35	2/5	
M	−5/2	−5/2		
−1 −3/2	4/7	3/7		
−2 −1/2	3/7	−4/7		
M	−7/2			
−2 −3/2	1			

2 × 2

m_1 m_2	J=4	J=3	J=2	J=1	J=0
M	+4				
+2 +2	1				
M	+3	+3			
+2 +1	1/2	1/2			
+1 +2	1/2	−1/2			
M	+2	+2	+2		
+2 0	3/14	1/2	2/7		
+1 +1	4/7	0	−3/7		
0 +2	3/14	−1/2	2/7		
M	+1	+1	+1	+1	
+2 −1	1/14	3/10	3/7	1/5	
+1 0	3/7	1/5	−1/14	−3/10	
0 +1	3/7	−1/5	−1/14	3/10	
−1 +2	1/14	−3/10	3/7	−1/5	
M	0	0	0	0	0
+2 −2	1/70	1/10	2/7	2/5	1/5
+1 −1	8/35	2/5	1/14	−1/10	−1/5
0 0	18/35	0	−2/7	0	1/5
−1 +1	8/35	−2/5	1/14	1/10	−1/5
−2 +2	1/70	−1/10	2/7	−2/5	1/5
M	−1	−1	−1	−1	
+1 −2	1/14	3/10	3/7	1/5	
0 −1	3/7	1/5	−1/14	−3/10	
−1 0	3/7	−1/5	−1/14	3/10	
−2 +1	1/14	−3/10	3/7	−1/5	
M	−2	−2	−2		
0 −2	3/14	1/2	2/7		
−1 −1	4/7	0	−3/7		
−2 0	3/14	−1/2	2/7		
M	−3	−3			
−1 −2	1/2	1/2			
−2 −1	1/2	−1/2			
M	−4				
−2 −2	1				

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2 \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1) \qquad d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Sign convention is that of Wigner (Group Theory, Academic Press, New York, 1959), also used by Condon and Shortley (The Theory of Atomic Spectra, Cambridge Univ. Press, New York, 1953), Rose (Elementary Theory of Angular Momentum, Wiley, New York, 1957), and Cohen (Tables of the Clebsch-Gordan Coefficients, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The signs and numbers in the current tables have been calculated by computer programs written independently by Cohen and at LBL. (Table extended April 1974.)

Physics Qualifying Examination UCI May 17, 1976

THERMODYNAMICS
AND
STATISTICAL MECHANICS

Do all 4 problems.

1. The force constant of a delicate spring balance is K $K = 0.5$ N/m $(F = -Kx)$. Because of the bombardment by air molecules, an object supported by the spring executes a random vibration.
   a) What is the average energy of this random motion?
   b) What is the rms displacement of the object away from its equilibrium position at room temperature?
   c) The spring balance is used to weigh an object whose mass is approximately 1 gram. What fractional uncertainty in its weight is contributed by the thermal fluctuations?

2. a) Show that for a photon gas, the number of photons depends on the temperature. Derive the form of this equation.
   b) Show that the average energy per photon is proportional to kT, i.e., similar to the classical ideal gas problem. Derive the form of this equation.

3. In a fission reactor operating at $600^{\circ}K$, the free neutron flux near its center is 4×10^{24} neutrons $m^{-2} - sec^{-1}$.
   a) Find the number of neutrons per cubic meter.
   b) Show that, to a good approximation, one can treat this system of Fermi-Dirac particles with Maxwell-Boltzman statistics.
   c) What is the pressure generated by this neutron gas?
   d) Derive an equation for the Fermi Energy (Chemical Potential) of this gas, and calculate its value.

4. Consider a very long (infinite) hollow pipe of cross sectional area A standing vertically on the surface of the earth. Assume that the gas within the pipe is an ideal monatomic gas of mass m at a constant temperature T.

   a) What is the partition function for this gas?

   b) What is the average energy per molecule?

   c) Does this gas obey the equipartition theorem? Explain your answer.